Fisher Discriminant Analysis - Overview

Huyen Do

April 20, 2010

1 Fisher Discriminant Analysis

Maximize $J(w) = \frac{w^T S_B w}{w^T S_W w}$ where S_B - between classes scatter matrix and S_W - within classes scatter matrix. $S_B = \sum_c (\boldsymbol{m}_c - \bar{x})(\boldsymbol{m}_c - \bar{x})^T$ $S_W = \sum_c \sum_{i \in c} (x_i - \boldsymbol{m}_c)(x_i - \boldsymbol{m}_c)^T$

J is invariant w.r.t rescalings if the vector $w \to \alpha w$. Hence, we can always choose w such that the denominator is simply $w^T S_W w = 1$, since it is a scaler itself. For this reason we can transform the problem of maximizing J in to the following constrained optimization problem

$$\min_w -\frac{1}{2} w^T S_B w s.t w^T S_B w = 1$$

The Lagrangian

$$L = -\frac{1}{2} w^T S_B w + \frac{1}{2} \lambda (w^T S_B w - 1)$$

The solution

$$S_B w = \lambda S_W w$$

or

$$S_W^- 1 S_B w = \lambda w$$

This almost looks like and eigen value equation, if the matrix $S_W^- 1 S_B$ would have been symmetric.

Solution: $w = S_W^{-1}(m_1 - m_2)$